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BOW WAVES BEFORE BLUNT SHIPS

By

G. Dagan and M. P. Tulin

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Hydronautics, Incorporated
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NOTATION

Dotted variables are dimensional; undotted variables are dimensionless

a, b	constants
B'	ship beam
D'	drag force
f, F	outer and inner complex potentials
$Fr_T = U'^2/gT'$ $Fr_L = U'^2/gL'$	Froude numbers
g	gravity acceleration
h'(x',z'), h'(x')	functions describing the hull shape
k(z)	function of complex variable (k = w + if)
L'	shiplength
L_1'	forebody length
N	free surface elevation (inner, dimensionless)
p'	pressure
P	pressure (inner, dimensionless)
q	velocity modulus
T'	draft
u', v'	velocity components
w'	velocity component (Sect. II), complex velocity w' = u' - iv' (Sect. III)
-U'	unperturbed velocity at infinity
U, V, W	velocity components (inner dimensionless Sect. II); w = u - iv (Sect. III)
$\vec{V}$	velocity vector
x', y'	coordinates

z'	coordinate (Sect. II), complex variable $z' = x' + iy'$ (Sect. III)
X, Y, Z	coordinates (inner, dimensionless)
γ	constant
Δ'	jet thickness ($\Delta = \Delta'/T'$)
$\epsilon = T'/L'$	draft/length ratio
$\epsilon_B = B'/L'$	beam/length ratio
$\epsilon^* = T'/gU'^2$	
ϕ'	velocity potential
Φ	velocity potential (inner dimensionless)
ω, ζ	auxiliary complex variables
ξ, μ	auxiliary variables
λ	angle and also dummy variable
θ	angle between velocity vector and x axis
$\Omega = -\ln (U-iV)$	

I. INTRODUCTION

The wave pattern created by a ship moving steadily in an ideal fluid and the related wave resistance are classical subjects of hydrodynamics. Although the theory has diversified and computational refinements have been achieved with time, there has been little essential progress beyond the linearized techniques introduced by Michell and Havelock. In essence the present method of solution of the gravity flow problem is based on two approximations: (i) the free surface condition is linearized and (ii) the hull is replaced by a singularity distribution along a line or a plane. The wave resistance is generally determined from the rate of energy radiated far away from the ship.

The above two basic approximations have been given foundation in a rational way in the last years by the application of the method of matched asymptotic expansions (Tuck 1965, Ogilvie 1967). It has been shown that the classical theory is in fact a first order term of an outer expansion in which the observer is fixed with respect to the ship length while the ratio draft/length or beam/length (or both) tend to zero and the Fr number based on length remains constant. In the vicinity of the body, in the inner zone, the solution is still valid, provided that the slenderness parameter is sufficiently small and the ship has a fine form. The latter condition has been somehow overlooked when applying the theory to actual ships which do not generally have a needle-like or knife-like shape. In the extreme case of a blunt shape there is stagnation at the bow and the linearized

assumptions are badly violated there: the speed and the free surface rise are no longer small perturbations of the uniform speed and of the horizontal level, respectively. At the stern the situation is different due to separation and viscous effects.

Realizing the importance of bluntness effects on resistance of real ships, we have initiated a study of the free surface flow near the bow and of the related resistance.

The present report summarizes our first results which, because of the complexity of the problem, involve in this initial stage rather crude approximations obtained for highly schematized configurations. It is our feeling, however, that this initial step yields basic understanding of the problem. We hope to be able to extend and refine the results in the future, to compare them with experiments, and eventually to apply them to actual ships.

II. INNER AND OUTER EXPANSIONS AND CLASSIFICATION OF SHIPS

1. Notation and Basic Equations

The symbols used in this report are given in "Notations" and also shown in part in Figure 1.

The ship is defined by the shape of its hull represented by the equation

$$f(x', y', z') = 0 \qquad [2.1]$$

or in one of the explicit forms

$$y' = h'(x', z') \qquad [2.2]$$

The three basic lengths associated with the hull are B', T' and L'. Additional geometrical coefficients or lengths may be considered, like the forebody length L_1'. The latter is important in characterizing the bluntness.

Assuming that the flow is steady and uniform at infinity, the equations satisifed by $\overline{V}'$ and η', given here for convenience of reference, are as follows:

$$\left.\begin{array}{l} \text{rot } \overline{V}' = 0 \\ \text{div } \overline{V}' = 0 \end{array}\right\} \text{(in the flow domain)} \qquad \begin{array}{l}[2.3 \\ [2.4]\end{array}$$

$$\left.\begin{array}{l} \dfrac{u'^2 + v'^2 + w'^2}{2} + g\eta' = \dfrac{U'^2}{2} \\ v' - u'\eta'_{,x'} - w'\eta'_{,z'} = 0 \end{array}\right\} \begin{array}{l}\text{(on the free-surface} \\ y' = \eta'(x',z'))\end{array} \qquad \begin{array}{l}[2.5] \\ [2.6]\end{array}$$

$$v' - u'h'_{,x'} - w'h'_{,z'} = 0 \qquad [2.7]$$

Equations [2.3] and [2.4] express as usual irrotationality and incompressibility, Equation [2.5] is the dynamic Bernoulli condition on the free surface, while Equations [2.6] and [2.7] are the kinematical boundary conditions along the free-surface and the hull respectively.

In order to render the solution unique, the radiation condition is imposed

$$u' = -U'; \; v' = w' = 0 \qquad (x' \to +\infty; \; y' \to -\infty) \qquad [2.8]$$

Equations [2.3] to [2.8] may be reformulated in terms of the velocity potential ϕ' by replacing $\overline{V}$ by grad ϕ'.

2. Outer Expansion; Classification of Hull Shapes

Economy is achieved by making the variables and the equations dimensionless in the standard way. Let us define the following outer variables

$$\vec{V} = \vec{V}'/U'(u,v,w = u'/U',v'/U',w'/U'); \; x,y,z = x'/L',y'/L',z'/L';$$

$$\eta = \eta'/L'; \; h = h'/L'; \; p = p'/\rho U'^2; \phi = \phi'/U'L'$$

[2.9]

The equations of flow [2.3] - [2.8] become, in terms of ϕ,

$$\nabla^2\phi = 0 \qquad \text{(in the flow domain)} \qquad [2.10]$$

$$\left.\begin{array}{ll} \dfrac{(\nabla\phi)^2}{2} + \eta/Fr_L^2 = 1/2 & [2.11] \\ \phi_{,y} - \phi_{,x}\eta_{,x} - \phi_{,z}\eta_{,z} = 0 & [2.12] \end{array}\right\} (y = \eta(x,z))$$

$$\phi_{,y} - \phi_{,x}h_{,x} - \phi_{,z}h_{,z} = 0 \quad (y = h(x,z)) \qquad [2.13]$$

$$u = -1; \; v = w = 0 \qquad (x \to +\infty; \; y \to -\infty) \qquad [2.14]$$

The solution of Φ depends on x, y, z and the parameters $Fr_L = U'/(gL')^{\frac{1}{2}}$, B'/L', T'/L' for a given hull shape. The classical technique for simplifying the nonlinear problem is to take advantage of the fact that B'/L' or T'/L' or both are much smaller than unity. With $\epsilon = T'/L'$ and $\epsilon_B = B'/L'$ an outer expansion is obtained by assuming that $\vec{V}$ and Φ may be expressed as a series associated with an asymptotic sequence based on ϵ or ϵ_B. This has been done in numerous publications (see for instance Wehausen and Laitone, 1960 and Tuck, 1965) and will not be repeated here. Since by definition

$$h\ (x,z) = \epsilon(x,z) \qquad [2.15]$$

where $H = O(1)$, it is natural to consider an expansion of the type

$$\Phi = -x + \epsilon\Phi_1 + O(\epsilon)$$

$$\eta = \epsilon\eta_1 + O(\epsilon) \qquad [2.16]$$

with x, y, z = O(1). For an outer observer with a position fixed with respect to the ship length, the ship collapses in a line or a plane at zero order and the flow is unperturbed. At first order (and we consider here only first order terms) the equations become the well known linearized equations

$$\nabla^2\phi_1 = 0 \qquad y < 0 \qquad [2.17]$$

$$\left.\begin{aligned} -u_1 + \eta_1/Fr_L^2 &= 0 \qquad [2.18] \\ v_1 - \eta_{1,x} &= 0 \qquad [2.19] \end{aligned}\right\} \quad (y = 0)$$

$$v_1 + H_{,x} = 0 \qquad \text{(on the hull)} \qquad [2.20]$$

$$u = v_1 = w_1 = 0 \qquad (x \rightarrow +\infty ; y \rightarrow -\infty) \qquad [2.21]$$

The hull, at first order, degenerates into:

(i) a line in the case of slender ships ($\epsilon_B = O(\epsilon)$),
(ii) a vertical plane in the case of thin ships ($\epsilon = O(1)$ and $\epsilon_B = O(1)$) and
(iii) a horizontal plane at $y = 0$ for flat ships ($\epsilon_B = O(1)$ and $\epsilon = O(1)$).

Different flow regimes, and equations accordingly, are obtained corresponding to the relationship between Fr_L and ϵ. Ogilvie (1967) has analyzed these possibilities. Since we concentrate here on displacement ships mainly, we should consider the following two possibilities:

(i) Small Fr_L number, $Fr_L^2 = O(1)$. In this case a direct expansion of Equations [2.10] - [2.11] gives

$$v_1 = 0 \qquad (y = 0) \qquad [2.22]$$

i.e., a rigid wall condition of the free surface. This expansion is discussed in detail in Section III.3.

(ii) $Fr_L = 0(1)$, which yields the ordinary ship resistance problem, with gravity waves left behind the ship (Equations [2.17] - [2.21])

Higher Fr_L lead to planing problems not considered here.

Problem (ii), by far the most interesting, has been solved by replacing the degenerated hull by: (i) a line of sources for slender ships, (ii) a source distribution in the mid-plane for thin ships and (iii) a pressure distribution on the free surface for flat ships (Lunde, 1952).

The fulfillment of the free-surface conditions [2.18] and [2.19] is equivalent to the extension of the flow in the whole space (above $y = 0$) and the introduction of an infinite system of singularities in $y > 0$, reflection of the ship singularities. In the case of small Fr_L just one image is sufficient in order to satisfy Equation [2.22].

3. <u>Inner Expansion; Bow Singularity</u>

The outer expansion is singular near the body in the $Fr_L = 0(1)$ case since the first order velocity tends there to infinity. Tuck (1965) has considered an inner expansion for slender ships. The inner variables are defined as

$$X = x = x'/L'; \; Y = y/\epsilon = y'/T'; \; Z = z/\epsilon = z'/T';$$

$$N = \eta/\epsilon = \eta'/T'; \; U, \; V, \; W = u, \; v, \; w \qquad [2.23]$$

The equations of flow [2.10] and [2.11] are again expanded by assuming that U,V,W and N are asymptotic series with respect to ϵ

$$U,V,W = U_o,V_o,W_o + O(\epsilon)$$

$$N = N_o + O(\epsilon) \qquad [2.24]$$

while $X,Y,Z = O(1)$.

In the inner limit the observer is fixed with respect to the beam (or draft). For such an observer, when $\epsilon \rightarrow 0$ the hull cross section keeps its shape unchanged while the shiplength tends to infinity. The equations become, at zero order, two-dimensional (in Y,Z) and the free surface condition becomes that of a rigid wall.

The matching of the outer and inner expansions yields (Tuck, 1965) the classical result, at first order : i.e. the replacement of the ship by a source system. The source strength is proportional to the cross-section area variation. The inner expansion is valid only if this variation is gradual, i.e. for fine ships. The slender body expansion fails in the bow region if the ship has some bluntness, and there the inner problem is no more one of two-dimensional flow in the Y,Z plane, nor is the condition on the free surface one of a rigid wall. For this reason we should call the slender body expansions outer and inner <u>midbody</u> expansions, in order to stress their limitations.

In the case of a blunt-bow ship the appropriate inner variables in the bow region are

$$X,Y,Z = x/\epsilon, y/\epsilon, z/\epsilon; \; U,V,W = u,v,w; \; N = \eta/\epsilon; \; \Phi = \phi/\epsilon \qquad [2.25]$$

$$U,V,W = U_o,V_o,W_o + O(\epsilon); \; \Phi = \Phi_o(X,Y,Z) + O(\epsilon); N = N_o + O(\epsilon) \qquad [2.26]$$

with $X,Y,Z = O(1)$.

While in the case of the inner midbody expansion the observer is fixed laterally with respect to the ship and at zero order the length tends to infinity, in both bow and stern directions, in the bow inner expansion the observer is fixed with respect to the bow and the shiplength tends to infinity sternwise.

Substituting [2.25] and [2.26] into Equations [2.10] - [2.13] we get at zero order

$$\nabla^2\Phi_o = 0 \quad \text{(in the flow domain)} \qquad [2.27]$$

$$\left.\begin{aligned} &\frac{U_o^2 + V_o^2 + W_o^2}{2} + \frac{1}{Fr_T^2} N_o = \frac{1}{2} \qquad [2.28] \\ &V_o - U_o N_{o,x} - W_o N_{o,z} = 0 \qquad [2.29] \end{aligned}\right\} (Z = N_o)$$

$$V_o - U_o H,_x - W_o H,_z = 0 \qquad (Z = H) \qquad [2.30]$$

The condition at infinity [2.14] is lost and is replaced by the matching with the outer expansion.

The bow Froude number $Fr_T = U'/(gT')^{\frac{1}{2}}$ is related to Fr_L and ϵ through

$$Fr_T^{\,2} = Fr_L^{\,2}/\epsilon \qquad [2.31]$$

Consequently the Bernoulli Equation [2.28] may have the following form depending on the order of magnitude of Fr_L:

(i) $Fr_L^{\,2} = O(1)(1/Fr_T^{\,2} = O(\epsilon))$. Then Equation [2.28]

$$U_o^{\,2} + V_o^{\,2} + W_o^{\,2} = 1 \qquad (Z = N_o) \qquad [2.32]$$

i.e. free gravity flow at zero order in the inner region.

(ii) $Fr_L^{\,2} = O(\epsilon)(1/Fr_T^{\,2} = O(1))$. Equation [2.28] remains unchanged and we have the full nonlinear gravity problem.

(iii) $Fr_L^{\,2} = O(\epsilon^2)(1/Fr_T^{\,2} = O(1/\epsilon))$. This case reduces to that of a rigid wall condition discussed in the preceding section.

In the case of slender and thin ships the flow in the vicinity of the bow is three-dimensional in all the above approximations. Further simplifications are achieved in the case of flat ships. Then the proper inner variables are

$$X = x/\epsilon;\ Y = y/\epsilon;\ Z = z;\ N = \eta/\epsilon;\ \Phi = \phi/\epsilon;\ U,V,W = u,v,w \qquad [2.33]$$

while the asymptotic expansion starts as

$$\Phi = \Phi_o + O(\epsilon);\ N = N_o + O(\epsilon);\ U,V = U_o,V_o + O(\epsilon);\ W = O(\epsilon) \qquad [2.34]$$

Again the substitution of [2.33] and [2.34] into Equations [2.10] - [2.14] gives, at zero order

$$\nabla_{x,y}{}^2\Phi_o = 0 \qquad [2.35]$$

$$\left.\begin{aligned} &\frac{U_o{}^2 + V_o{}^2}{2} + \frac{1}{Fr_T{}^2} N_o = \frac{1}{2} && [2.36] \\ &V_o - U_o N_{o,x} = 0 && [2.37] \end{aligned}\right\} (Y = N_o)$$

$$V_o - U_o H,_x = 0 \qquad (Y = H) \qquad [2.38]$$

and the problem is reduced to that of gravity flow in the vertical X,Y plane in the vicinity of a body of shape Y = H(X,Z) (here Z appears as a parameter). The requirement of flatness has the meaning of T'/B' < 1, but still allows for B'/L' < 1. In the flat ship approximation the observer attached to the bow sees both width and length tending to infinity (although possibly at different rates). Obviously this approximation is not valid near corners or regions of large change of H with X. There the full three-dimensional flow or some other approximations have to be considered. Again we obtain the three different cases discussed above depending on whether $1/Fr_T{}^2 = O(\epsilon), O(1)$ or $O(1/\epsilon)$.

We summarize the discussion of all the encountered cases in the following table:

DISPLACEMENT SHIPS ($Fr_L < 1$)

<table>
<tr><td rowspan="2">$Fr_L^2 = U'^2/gL'$</td><td colspan="2">$O(1)$</td><td rowspan="2">$O(1)$</td></tr>
<tr><td>$O(\epsilon^2)$</td><td>$O(\epsilon)$</td></tr>
<tr><td>$Fr_T^2 = U'^2/gT'$
$= Fr_L^2/\epsilon$</td><td>$O(\epsilon)$</td><td>$O(1)$</td><td>$O(\epsilon^{-1})$</td></tr>
<tr><td>Outer Expansion</td><td rowspan="3"><u>Rigid Wall Condition Everywhere</u>
The un-perturbed flow is the state of rest.</td><td><u>Rigid Wall Condition</u></td><td><u>Linearized gravity waves far from the ship</u>. The ship is replaced by a line (slender) or a plane (thin and flat) distribution of singularities.</td></tr>
<tr><td>Inner Midbody Expansion</td><td><u>Rigid Wall Condition</u></td><td><u>Rigid Wall Condition</u>
Slender ships: two-dim. flow in vertical planes normal to the centerline (Tuck, 1965).</td></tr>
<tr><td>Inner Bow Expansion</td><td><u>Nonlinear Gravity Flow</u>. Slender and thin ships: three-dim. flow; flat ships: two-dim. flow in vertical planes normal to the bow.</td><td><u>Nonlinear free-gravity flow</u>
Slender ships: three-dim. flow.
Thin ships: three-dim. flow near a strut.
Flat ship: two-dim. flow in vertical planes normal to a body of infinite length.</td></tr>
</table>

obs.: In the case of fine ships the midbody expansion is valid everywhere.

Finally we present in Figure 2 a plot of B'/T' and Fr_T for more than one hundred existing ships.

There is no apparent correlation between the two parameters. At any rate most of the ships considered are flat rather than thin $(B'/T' = 2.2 \div 3.4)$.

The draft Froude number Fr_T is of order one in most cases, but reaches values as high as 2 for a rapid containership and more than 3 for cruisers and destroyers.

III. GRAVITY FLOW PAST TWO-DIMENSIONAL BLUNT BODIES OF SEMI-INFINITE LENGTH

1. General

In the preceding sections it was shown that in the case of flat ships the inner bow flow reduces to a two-dimensional flow in a vertical plane normal to the bow. In the remaining sections of this work we consider exclusively such flows. Moreover, we are assuming that the outer flow is also two-dimensional and that the body is of semi-infinite length. Obviously, these assumptions simplify the problem considerably. The essential features of the bow flow are, nevertheless, included in the picture. We plan to apply the results by some approximate techniques to actual ships in the future, taking advantage of the fact that for most ships the ratio draft/beam is smaller than unity.

Consistent with the range of Fr_L considered, which apply to displacement ships, we assume that the bottom of the midbody is horizontal. The results permit however, to compute trim to a first approximation, but we do not consider this problem here.

In the case of a two-dimensional flow (Figure 3) the dimensionless velocity potential depends on only one parameter for a given hull shape: $\Phi = \Phi\ (x,y;Fr_T)$. Consequently, the possible asymptotic expansions of the exact Equations [2.10] - [2.14], with the z components deleted, reduce to the following cases:

(i) Small Fr_T . In this case the Bernoulli equation gives a rigid wall condition in a first approximation. A uniform expansion solves the problem. Results for the first and second order approximations are given in Section III.3.

(ii) Large Fr_T. In this case the outer flow conforms to equations similar to the linearized Equations [2.17]-[2.21], while the inner flow is that of a free-gravity flow at zero order. For this regime we suggest two possible inner flow models: The jet model discussed in detail in Section III.4 and the spiral vortex model. It is presumed that the jet model is adequate for large Fr_T, while the spiral vortex model represents moderate to large Fr_T flows. Only the theory for the former is presented herein.

In addition a discussion of the exact equations of free-surface gravity flow near a stagnation point is presented in Section III.2.

2. Free-Surface Gravity Flow near a Stagnation Point

Let us consider the confluence between a free-surface and a rigid wall in the vicinity of a stagnation point (Figure 4a).

In the symmetrical case ($\lambda_1 = \pi - \lambda_2$) the classical Stokes result (Wehausen and Laitone, 1960) requires that $\lambda = \lambda_2 - \lambda_1 = 120^{\circ}$. This result will now be extended for other possible angles between AO and OB.

In the vicinity of O(z = 0, Figure 5a) we assume that the z-plane is mapped on the complex potential plane f (Figure 5b) by

$$z = a\,e^{-i\lambda_1} f^{\lambda/\pi} + R(f) \qquad [3.1]$$

AOB being obviously a stream line.

The function R, which has to vanish at O, is assumed to be in the vicinity of O of the form

$$R = b\,f^{\gamma} \qquad [3.2]$$

with $b = b' e^{i\delta}$ a complex number and γ a real number.

Obviously, $\gamma > \lambda/\pi$, otherwise the mapping of the corner AOB is not ensured.

In order to apply the Bernoulli equation along AO let us determine y and $q^2 = u^2 + v^2$ as functions of ϕ. From Equations [3.1] and [3.2] we obtain on AO ($f = \phi$)

$$y = -a \sin \lambda_1\, \phi^{\lambda/\pi} + b' \sin \delta\, \phi^{\gamma} \qquad [3.3]$$

$$1/q^2 = x,_\phi^{\,2} + y,_\phi^{\,2} = \left(\frac{a\lambda}{\pi}\right)^2 \phi^{2(\lambda/\pi-1)} + 2\,\frac{ab'\lambda\gamma}{\pi}\cos(\lambda_1+\delta)\phi^{(\lambda/\pi-\gamma-2)} \qquad [3.4]$$

By expanding Equation [3.4], q^2 is found as

$$q^2 = \left(\frac{\pi}{\lambda a}\right)^2 \phi^{-2(\lambda/\pi-1)}\left[1 - 2\,\frac{\pi b'\gamma}{a\lambda}\,\phi^{\gamma-\lambda/\pi}\cos(\lambda_1+\delta) + \ldots\right] \qquad [3.5]$$

Substituting y and q^2 into Bernoulli's equation and retaining terms of order $\phi^{(2-\lambda/\pi)}$ or ϕ^2 at most we get

$$y + \frac{q^2}{2g} = -\,a \sin\lambda_1\phi^{\lambda/\pi} + b'\sin\delta\;\phi^{\gamma} + \frac{1}{2g}\left(\frac{\pi}{\lambda a}\right)^2\phi^{-2(\lambda/\pi-1)} + \ldots \equiv 0 \qquad [3.6]$$

The identity [3.6] yields the following relationships between λ_1 and λ:

(i) $\lambda_1 \neq 0$. The first two terms of [3.6] give

$$\lambda = 2\pi/3$$
$$-\frac{g}{8}\,\frac{1}{a^3} = g \sin\lambda_1 \qquad [3.7]$$

This is Stokes classical result. Obviously, $\lambda_2 > 2\pi/3$

(ii) $\lambda_1 = 0$. The first term of Equation [3.6] vanishes, and the remaining give

$$\gamma = -\,2(\lambda/\pi - 1) \qquad [3.8]$$

Since $\gamma > \lambda/\pi$ Equation [3.8] shows that $\lambda < 2\pi/3$. A particular case is that of $\gamma = 1$, which renders the function R analytical. In this case Equation [3.8] gives $\lambda = \pi/2$, i.e. the confluence between a horizontal free-surface and a vertical wall.

In conclusion there are two possible angles between a free-surface and a rigid wall at the stagnation point:

(i) if the wall is inclined with respect to the horizontal at an angle larger than $120^\circ (2\pi/3 < \lambda_2 < \pi)$ the free surface intersects the wall at 120° $(\lambda = 2\pi/3)$ and,

(ii) if the wall is inclined at less than 120° $(\lambda < 2\pi/3)$ the free-surface is horizontal $(\lambda_1 = 0)$.

We will consider blunt bows of the latter type in Section III.3.

3. Small Froude Number Flow ($Fr_T < 1$)

(a) General

We consider here the flow past a blunt body of the shape of Figure 5a. An asymptotic expansion with Fr_T as a small parameter has as its zero order term the state of rest corresponding to $Fr_T = 0$. Hence, it is appropriate to make the variables dimensionless in the following way

$$X = x'/T'; Y = y'/T'; N = \eta'/T'; H = h'/T'; U = u'/(gT')^{1/2}$$

$$V = v'/(gT')^{1/2}; F = \Phi + i\Psi = (\phi' + i\psi')/g^{1/2}T'^{3/2}; P = p'/\rho g T' \qquad [3.9]$$

The exact boundary conditions of two-dimensional flow are:

$$\left.\begin{aligned} &\frac{U^2 + V^2}{2} + N = \frac{Fr_T^{\,2}}{2} \qquad [3.10] \\ &V - UN,_X = 0 \qquad [3.11] \end{aligned}\right\} \quad (Y = N(X))$$

$$V - UH,_X = 0 \qquad (Y = H(X)) \qquad [3.12]$$

$$V = 0, N = 0, U = -Fr_T \qquad (|Z| \rightarrow \infty) \qquad [3.13]$$

$W = U - iV$ and $F = \Phi + i\Psi$ being analytical functions of $Z = X + iY$.

(b) Small Perturbation Expansion

In order to simplify the nonlinear problem we seek a solution valid for small Fr_T. When $Fr_T \rightarrow 0$, while $X, Y = O(1)$ the flow tends to rest (Equation 3.13) while the body retains its shape, the thickness being equal to unity.

It is a matter of simple algebra to show that a nontrivial small perturbation expansion has the form

$$\left.\begin{aligned} U &= Fr_T\, U_1 + Fr_T^{\,3} U_2 + \ldots \\ V &= Fr_T V_1 + Fr_T^{\,3} V_2 + \ldots \\ N &= Fr_T^{\,2} N_1 + Fr_T^{\,4} N_2 + \ldots \end{aligned}\right\} \qquad [3.14]$$

The above expansion will be shown to be regular at infinity and consequently there is no need to consider inner and outer expansions separately.

Expanding $U(X,Y)$ and $V(X,Y)$ in the vicinity of $Y = N(X)$ as given by Equations [3.14] and substituting in Equations [3.10]-[3.13], we get the following set of equations after separating terms of the same order

(i) U_1, V_1, N_1

$$V_1 = 0 \qquad (X > 0,\ Y = 0) \qquad [3.15]$$

$$N_1 = \frac{1}{2}(1 - U_1^2) \qquad (X > 0,\ Y = 0) \qquad [3.16]$$

$$V_1 - U_1 H,_X = 0 \qquad (Y = H(X) \qquad [3.17]$$

$$V_1 = 0;\ U_1 = -1 \qquad (|Z| \rightarrow \infty) \qquad [3.18]$$

Hence the first order approximation is that of a rigid wall on the free surface and uniform flow at infinity.

(ii) U_2, V_2, N_2

$$V_2 = (U_1 N_1),_X \qquad (X \qquad = 0) \qquad [3.19]$$

$$N_2 = -U_1 U_2 \qquad (X > 0) \qquad [3.20]$$

$$V_2 - U_2 H,_X = 0 \qquad (Y = H(X)) \qquad [3.21]$$

$$V_2 = 0, \; U_2 = 0 \qquad (|Z| \to \infty) \qquad [3.22]$$

In the second order approximation, the condition on the unperturbed free surface (Equation 3.19) is equivalent to a distribution of sources generated by the first order flow, with no flow at infinity. It is easy to ascertain that

$$\int_0^\infty V_2 \, dX = N_1 V_1 \Big|_0^\infty = 0 \qquad [3.23]$$

Since $N_1 \to 0$ as $X \to \infty$ and $U_1 = 0$ at the stagnation point $X = 0$. Obviously W_1 and W_2 are analytical functions of Z.

Higher order terms satisfy equations similar to those of second order, but the computations become tedious as the order is increased.

c. General Solution

The solutions of the different order approximations may be obtained as follows (Figure 5): The region AOBA of the Z plane is mapped on the ζ half plane by

$$Z = Z(\zeta) \qquad [3.24]$$

and the first order complex potential $F_1 = \Phi_1 + i\Psi_1$ is mapped on the same ζ plane by

$$F_1 = \text{const} \times \zeta \qquad [3.25]$$

At second (and higher) order the imaginary part of $F_2 = \Phi_2 + i\Psi_2$ is given along the ξ axis (Figure 5b) by Equations [3.19] and [3.21]

$$\begin{aligned} \Psi_2 &= -U_1 N_1 \qquad (\xi > 1) \\ \Psi_2 &= \text{const} \qquad (\xi < 1) \end{aligned} \qquad [3.26]$$

and $F_2(\zeta)$ is found by solving the related Dirichlet problem.

(d) Application to the Rectangular Body

As a simple example we consider the box-like shape body of Figure 6. The AOBO region of the Z plane is mapped on the ζ half-plane by

$$Z = \frac{1}{\pi}(\zeta^2 - 1)^{\frac{1}{2}} + \frac{1}{\pi} \ln \left[\zeta + (\zeta^2 - 1)^{\frac{1}{2}}\right] \qquad [3.27]$$

where both $(\zeta^2 - 1)^{\frac{1}{2}}$ and the logarithm have real determination on $\xi > 1$.

With $F_1 = -\zeta/\pi$ we get

$$W_1 = U_1 - iV_1 = \frac{dF_1}{dZ} = -\left(\frac{\zeta-1}{\zeta+1}\right)^{\frac{1}{2}} \qquad [3.28]$$

In particular from Equation [3.16] N_1 is given by

$$N_1 = \frac{1}{2}(1 - U_1^2) = \frac{1}{\xi+1} \qquad (\xi > 1) \qquad [3.29]$$

Equations [3.29] and [3.27] describe the shape of the free surface in a parametric form. $N_1(X)$ is represented graphically in Figure 7.

The next order term $F_2 = \Phi_2 + i\Psi_2$ has, according to Equations [3.21], [3.28] and [3.29], the imaginary part

$$\Psi_2 = -U_1 N_1 = \frac{1}{\xi+1}\left(\frac{\xi-1}{\xi+1}\right)^{\frac{1}{2}} \qquad (\xi > 1,\ \mu = 0) \qquad [3.30]$$

$$\Psi_2 = 0 \qquad (\xi < 1,\ \mu = 0)$$

and W_2 vanishes at infinity.

$F_2(\zeta)$, with given imaginary part on the real axis ξ is determined by the Cauchy integral

$$F_2(\zeta) = -\int_1^\infty \Psi_2(\xi)\,\frac{d\xi}{\zeta-\xi} = -\frac{1}{\pi}\int_1^\infty \frac{(\xi-1)^{\frac{1}{2}}}{(\xi+1)^{3/2}}\,\frac{d\xi}{\zeta-\xi} \qquad [3.31]$$

The integration in Equation [3.31] may be carried cut analytically, the result being for $\zeta = \xi$, $\xi > 1$

$$\Phi_2(\xi) = \frac{1}{\pi}\left\{\frac{2}{1+\xi} + \left(\frac{\xi-1}{\xi+1}\right)^{\frac{1}{2}} \frac{\ln[\xi-(\xi^2-1)^{\frac{1}{2}}}{\xi+1}\right\} \qquad [3.32]$$

The velocity $U_2(0,\xi)$ is, accordingly

$$U_2(U_1\xi) = \frac{\partial\phi_2}{\partial\xi}\frac{\partial\xi}{\partial X} = -\frac{1}{(1+\xi)^2}\left\{3\left(\frac{\xi-1}{\xi+1}\right)^{\frac{1}{2}} + \frac{\xi-2}{\xi+1}\ln\left[\xi-(\xi^2-1)^{\frac{1}{2}}\right]\right\} \qquad [3.33]$$

and finally the free surface elevation is given by Equation [3.20]

$$N_2 = -U_1U_2 = -\frac{\xi-1}{(\xi+1)^3}\left\{3 + \frac{\xi-2}{(\xi^2-1)^{\frac{1}{2}}}\ln\left[\xi-(\xi^2-1)^{\frac{1}{2}}\right]\right\} \qquad [3.34]$$

N_2 and $N/Fr_T^2 = N_1 + Fr_T^2 N_2$ as functions of X are represented graphically in Figure 7.

(e) Pressure Distribution and Forces Acting on the Body

The dimensionless pressure has the following expansion resulting from the Bernoulli Equation,

$$P = -Y + \frac{Fr_T^2}{2}\left[1-(U_1^2 + V_1^2)\right] - Fr_T^4(U_1U_2 + V_1V_2) + \ldots \qquad [3.35]$$

A detailed analysis of the forces acting on the body show that the drag is equal to zero, as it should be in an ideal fluid flow with no waves. The dynamical vertical force as well as the moment are different from zero even at first order. The possibility of computing sinkage and trim via the small Fr_T expansion will be explored in a future work.

(f) Stability of the Free Surface

Experiments show that as Fr_T increases a breaking wave appears in front of the body (Baba, 1969). The inspection of Figure 7 reveals that as Fr_T increases the free surface becomes steeper. Because of the convexity of the free-surface near the body, the centrifugal effect diminishes the pressure gradient normal to the surface. When the pressure gradient becomes less than zero, the pressure at some point inside the fluid is smaller than the atmospheric pressure. As shown by G.I.Taylor (1950), such a condition leads to the instability of the free-surface and very often, to its disruption or breaking. Adopting the vanishing of the pressure gradient as a criterion for free-surface stability, i.e., the Taylor stability criterion, we are led to the condition

$$\frac{\partial p'}{\partial n'} = -\rho g(1 + \eta'^{2}_{,x'})^{-\frac{1}{2}} + \rho\,\frac{u'^2 + v'^2}{r'} = 0$$

Rewriting Equation [3.35] in dimensionless variables and with $r' = -\eta'_{,x'x'}\,(1+\eta'^{2}_{,x'})^{-3/2}$ we arrive at marginal stability for

$$(U^2 + V^2)N_{,XX}(1+N_{,X})^{-3/2} = -(1+N_{,X}^{2})^{-1/2} \qquad [3.36]$$

Expanding Equation [3.36] on the free-surface yields, at Fr_T^6 order,

$$\frac{\partial P}{\partial n} = - Fr_T{}^4 U_1{}^2 N_{,XX} - Fr_T{}^6 (U_1{}^2 N_{2,XX} + 2U_1U_2N_{1,XX})$$

$$- 1 + \frac{1}{2} Fr_T{}^4 N_{1,X}{}^2 + Fr_T{}^6 N_{1,X} N_{2,X} = 0 \qquad [3.37]$$

The stability criterion as expressed by Equation [3.37] has been applied to the flow plast a box shaped body. With N_1, N_2, U_1 and U_2 given in Equations [3.28], [3.29], [3.33] and [3.34] the different terms of [3.37] have been computed as functions of ξ. In Figure 7a we give the location of the point of minimum $-\partial P/\partial n$ as a function of Fr_T. The point of minimum $-\partial P/\partial n$ is located at $X \approx 0.3$.

(g) Discussion of Results

A uniformly valid expression for the velocity and free-surface profile has been derived. The solution of first order is based on a rigid wall approximation while in the second order a singularity distribution is used to satisfy the free-surface condition. The solution has the proper behaviour at the stagnation point S (Figure 7) since both W and dN/dX vanish there in the case of a blunt body. The behavior at infinity is also correct.

Inspection of the free-surface profile as a function of Fr_T(Figure 7) shows that as Fr_T increases the free-surface becomes steeper. This is a second order effect and reflects the influence of the nonlinearity of the free-surface condition. Although at Fr_T of order one or larger it is doubtful whether the first two terms represent the expansion accurately, the trend is nevertheless obvious.

The pressure gradient normal to the free-surface decreases with Fr_T (Figure 8). At $Fr_T \cong 1.5$, Taylor instability of the free surface occurs. Of course, the value of the critical Fr_T predicted by this second order theory is probably not too accurate, but the calculation serves to suggest the probable existence of a critical value of Fr_T, beyond which wave breaking occurs.

In analogy with progressive free surface waves, we might even expect the onset of Taylor Instability to coincide with the non-existence of a free surface wave without breaking.

The present approach permits an evaluation of the influence of the bow shape on the inception of free surface breaking as well as the determination of the sinkage and of the trim of bodies of finite length.

4. High Froude Number Flow ($Fr_T > 1$): The Jet Model

(a) General

In the case of high Fr_T it is appropriate to relate the variables in the outer zone to the outer length U'^2/g and the velocity U'

$$x = gx'/U'^2; \; y = gy'/U'^2; \; \eta = g\eta'/U'^2; \; h = gh'/U'^2;$$

$$u = u'/U'; \; v = v'/U'; \; \phi = g\phi'/U'^3 \qquad [3.38]$$

The exact boundary conditions become now

$$\frac{u^2 + v^2}{2} + \eta = \frac{1}{2} \qquad (y = \eta) \qquad [3.39]$$

$$u - v\eta,_x = 0 \qquad (y = \eta) \qquad [3.40]$$

$$u - vh,_x = 0 \qquad (y = h) \qquad [3.41]$$

$$u \rightarrow - 1;\ v \rightarrow 0 \qquad (x \rightarrow \infty;\ y \rightarrow -\infty) \qquad [3.42]$$

with $w = u - iv$ an analytical function of $z = x + iy$.

At the limit $T'g/U'^2 \rightarrow (Fr_T{}^2 \rightarrow \infty)$ the body collapses into a line along $y = 0$ (Figure 3) and the unperturbed state is that of uniform flow. The first order equations are the linearized equations of gravity waves of the type [2.18] - [2.19] (see next paragraph).

The problem of two dimensional flow, in this approximation, has been studied extensively. For the case of a blunt body at the free-surface two types of representations have been suggested in the literature:

(i) The replacement of the body by a source (Wehausen and Laitone, 1960) or by a constant pressure acting on the free-surface behind the bow (Lunde, 1952). It is easy to ascertain that the two are identical if the source is located on $y = 0$.

The first order velocity potential for a source of strength Q is (Wehausen and Laitone, 1960)

$$f_1(z) = \phi_1 + i\Psi_1 = \frac{Q}{\pi} \ln z - \frac{Q}{\pi} e^{-iz} \int_{\infty}^{z} \frac{e^{i\lambda}}{\lambda} d\lambda \qquad [3.43]$$

The free-surface profile corresponding to this solution has a wavy shape far behind the origin. It cannot, therefore, simulate a semi-infinite body of arbitrary shape. Near the origin, an expansion of $f_1(z)$ for small z shows that the free-surface is continuous there, since the integral in [3.38] behaves like $\ln z$ for small z. The complex velocity is singular near the origin like $\ln z$. This behavior will be found unsatisfactory for matching with the inner solution (paragraphs c, e).

(ii) The replacement of the body by a pressure distribution singular at the leading edge like $|x|^{-\frac{1}{2}}$. This approach is used in studies of planing surfaces (Sedov 1965, Maruo 1951, Squire 1957). Approximate solutions have been found for inclined flat plates of finite length by a Fourier series expansion of the pressure distribution. In these solutions the velocity w is singular near the leading edge (z = 0) like $z^{-\frac{1}{2}}$, while the free-surface is continuous there. For this reason this type of singularity, although stronger than that of (i), is still too weak in order to permit matching with the inner expansion (paragraphs c, e). An interesting feature of the planing solution is the fact that the pressure distribution is integrable. For this reason the leading edge correction and the inner expansion are not essential. It was nevertheless assumed that a jet exists at the leading edge and Wagner (1932) has linked the jet flow and the pressure singularity in a way similar to the matching of the inner and outer expansions.

Wu (1967) has studied the flow past an inclined surface in the high Fr_1 regime by matched asymptotic expansion.

In the following paragraphs we study the flow past a blunt semi-infinite body. The outer expansions corresponds to a regime in which $Fr_T > 1$ while $Fr_L \to 0$. Hence we are in the range of displacement ships, the buoyancy being much larger than the dynamic lift, dynamic effects being important only in the bow region.

(b) The Inner and Outer Expansions

The inner and outer expansions of Equations [3.39] - [3.42] follow closely the derivations of Section II.2, the body length being now immaterial.

The outer expansion has the form [2.16], with $\epsilon = \epsilon^* = 1/Fr_T^{\,2} = T'g/U'^2$ this time. Again the choice of the first order expansion is dictated by the fact that

$$h(x) = \epsilon^* H(x) \text{ and } H(x) = O(1) \qquad [3.44]$$

The first order terms satisfy equations similar to [2.17] - [2.21] which may be written in a complex form as

$$\mathrm{Re}\,(w_1 + if_1) = 0 \qquad (x > 0,\ y = 0) \qquad [3.45]$$

$$\Psi_1 = \eta_1 \qquad (x > 0,\ y = 0) \qquad [3.46]$$

$$\Psi_1 = H \qquad (x < 0, \; y = 0) \qquad [3.47]$$

$$w_1 = 0 \qquad (x \to +\infty) \qquad [3.48]$$

$f_1 = \Phi_1 + i\Psi_1$ and $w_1 = u_1 - iv_1$ being analytical functions of z in the domain $y < 0$.

The inner variables are those of [2.33] with Z and W deleted. The zero order inner expansions are exactly the free-gravity flow Equations [2.35] - [2.38].

(c) The Zero Order Inner Solution for the Rectangular Body

Let us consider again the simple case of a rectangular body (Figure 9). In the inner limit X and Y are fixed with respect to the bow and we assume that the free-gravity flow there takes the form of a jet directed upwards. Gravity effects are taken into account along the free-surface upstream by the outer expansion. The same effect on the jet upwards at some distance from the bow is ignored.

The solution of the inner problem follows the classical methods of free streamline flow studies (Gurevich, 1965).

The complex potential F_o plane is mapped on the auxiliary $\zeta = \xi + i\mu$ half-plane by

$$\frac{dF_o}{d\zeta} = -\frac{\Delta}{\pi}\frac{\zeta+1}{\zeta} \qquad [3.49]$$

The function $\Omega_o = \ell n\ (1/W_o) = \ell n\ (1/|W_o|) + i\theta$ has given imaginary and real parts on the boundaries

$$\left.\begin{array}{ll} \mathrm{Re}\Omega_o = 0 & (AJ; \xi > 0) \\ \theta = \pi/2 & (SJ;\ -1 < \xi < 0) \\ \theta = 3\pi/2 & (SB;\ -b^2 < \xi < -1) \\ \theta = \pi & (BA;\ \xi < -b^2) \end{array}\right\} \qquad [3.50]$$

where

b^2 is an arbitrary constant.

The mapping of Ω_o on ζ is a solution of a classical mixed problem (Signorini problem) which is reduced to a Dirichlet problem for the function $\Omega_o/F_o^{\frac{1}{2}}$. The result of the integration of Cauchy's integrals is

$$\Omega_o = \ell n \left[\frac{\zeta^{\frac{1}{2}} - i}{\zeta^{\frac{1}{2}} + i} \left(\frac{\zeta^{\frac{1}{2}} + ib}{\zeta^{\frac{1}{2}} - ib}\right)^{\frac{1}{2}}\right] + i\pi \qquad [3.51]$$

With a new auxiliary plane ω related to ζ through

$$\omega = \zeta^{\frac{1}{2}} \qquad [3.52]$$

we get from [3.51] and [3.52]

$$W_o = -\frac{\omega+i}{\omega-i}\left(\frac{\omega-ib}{\omega+ib}\right)^{\frac{1}{2}} \qquad [3.53]$$

Eigensolutions of the type $\Omega_o \sim i\zeta^{n+1/2}$ are ruled out since they yield infinite or zero velocity far downstream.

The mapping of the Z plane on the ω plane results from the basic relationship

$$dZ = \frac{dF_o}{W_o} \qquad [3.54]$$

Substituting [3.49], [3.52] and [3.53] into Equation [3.54] and taking in consideration that $Z = -i$ for $\omega = -ib$ (Figure 8) we get

$$Z + i = \frac{2\Delta}{\pi} \int_{-ib}^{\omega} \frac{\omega^2 H}{\omega} \frac{\omega - i}{\omega + i} \left(\frac{\omega + ib}{\omega - ib} \right)^{\frac{1}{2}} d\omega \qquad [3.55]$$

The integral of [3.55] can be carried out in a closed form with the result

$$Z + i = \frac{2\Delta}{\pi} \left\{ \frac{\omega}{2} (\omega^2 + b^2)^{\frac{1}{2}} - \frac{b^2}{2} \ln \frac{(\omega^2 + b^2) + \omega}{-ib} - i(2-b)(\omega^2 + b^2)^{\frac{1}{2}} \right.$$
$$\left. + (2b-1) \ln \frac{(\omega^2 + b^2) + \omega}{-ib} + i \ln \frac{b + (\omega^2 + b^2)^{\frac{1}{2}}}{i\omega} \right\} \qquad [3.56]$$

In all the above formulae the square roots and the logarithm have real determination on ω = real.

The unknown constants Δ and b have to be determined from matching with the outer expansion. For this purpose let us seek the behavior of Z and W_o far from the bow, i.e. for $|\omega| \gg b$. From Equation [3.56] we get for large ω

$$Z + i = \frac{2\Delta}{\pi}\left[\frac{\omega^2}{2} - i(2-b)\,\omega + (2b-1-\frac{b^2}{2})\,\ell n\,\omega + i\,\frac{\pi}{2}\,(2b-1-\frac{b^2}{2}) + \frac{ib}{\omega} + \ldots\right] \qquad [3.57]$$

Similarly, from Equation [3.53] we obtain for W_o

$$W_o = -1 - \frac{i(2-b)}{\omega} + \frac{(b-2)^2}{2\omega^2} - \frac{i(b^3-2)}{3\omega^3} + \ldots \qquad [3.58]$$

Two cases of interest are to be discussed separately:

(i) $b \neq 2$. In this case in a first approximation

$$W_o = -1 - \frac{i(2-b)\Delta^{\frac{1}{2}}}{\pi^{\frac{1}{2}}}\,\frac{1}{Z^{\frac{1}{2}}} + \ldots \qquad [3.59]$$

$$N_o = -1 + (2b-1-\frac{b^2}{2})\Delta - 2(2-b)\sqrt{\frac{\Delta}{\pi}}\;X^{\frac{1}{2}} + \ldots .\,(X > 0) \qquad [3.60]$$

$$N_o = -1 \qquad (X < 0) \qquad [3.61]$$

Hence, the velocity perturbation behaves like $Z^{-\frac{1}{2}}$, while $N_o \sim -X^{\frac{1}{2}}$.

(ii) $b = 2$. For this distinguished value

$$W_o = -1 - i\,\frac{2\Delta^{3/2}}{\pi^{3/2}}\,\frac{1}{Z^{3/2}} + \ldots . \qquad [3.62]$$

$$N_o = -1 + \Delta + \frac{4\Delta^{3/2}}{\pi^{3/2}} \frac{1}{X^{1/2}} \qquad (X > 0) \qquad [3.63]$$

$$N_o = -1 \qquad (X < 0) \qquad [3.64]$$

In this case for large X the velocity perturbation decays like $Z^{-3/2}$, while N_o tends to the constant value $-1 + \Delta$ like $X^{-1/2}$.

There is no major difficulty in determining the zero inner solution for bodies of other shapes than the rectangular, provided that θ is given as a function of ξ (for instance, a polygonal body). In the case of an arbitrary body with given θ as a function of x (or y) the problem becomes extremely difficult and leads to an integral equation for $\theta(\xi)$ (Wu, 1967).

(d) The First Order Outer Solution

The outer problem reduces to the determination of $f_1(z)$ subject to conditions [3.45], [3.46] and [3.48], while for the particular case of a rectangular body Equation [3.47] gives

$$\Psi_1 = 1 \qquad (x < 0,\ y = 0) \qquad [3.65]$$

The problem is made unique if the singular behavior of f_1 (or w_1) near the bow ($z = 0$) is prescribed. The inner solution shows that there exist two possibilities for W_o: Equation [3.59] or Equation [3.62]. A detailed study shows that matching is possible only in the second case. The reason is the following: if $b \neq 2$ the matching requires that $w_1 \sim z^{-\frac{1}{2}}$ near

the origin and Δ or (2-b) has to be of the order $\epsilon^{*\frac{1}{2}}$. In the first case η_1 is continuous at $x = 0$ and has the value $\eta_1 = -1$ there; this requires a solution with η_1 dropping from $\eta_1 = 0 (x = \infty)$ to $\eta_1 = -1$ $(x = 0)$. Such a solution is not possible for the assumed type of singularity of w_1. If (2-b) is $O(\epsilon^{*\frac{1}{2}})$ and $\Delta = O(1)$, η_1 has a jump at $x = 0$ from $(-1 + \Delta)$ to (-1). Again the assumed type of singularity of w_1 does not allow for a discontinuous η_1 (see paragraph a of this section).

Consequently, we adopt the value $b = 2$, and the inner term contains Δ as the only unknown. Moreover, w_1 behaves near $z = 0$ like $z^{-3/2}$ while η_1 is singular like $x^{-\frac{1}{2}}$ for $x > 0$.

An exact solution of $f_1(z)$ is still difficult. The usual way to find it (Sedov, 1965) is to consider the function $w_1 + if_1$ (suggested apparently for the first time by Keldish) and to continue it analytically over $x > 0$ in the entire z plane cut along $x < 0$. With

$$w_1(z) + if_1(z) = k(z) \qquad [3.66]$$

the unknown function $k(z)$ has to be imaginary for $z = x > 0$. Its real part is in fact the linearized pressure. The solution of Equation [3.66] with the radiation condition [3.48] is

$$f_1(z) = e^{-iz} \int_{\infty}^{z} e^{i\lambda} k(\lambda) d\lambda \qquad [3.67]$$

Finally $k(\lambda)$ has to be determined from Equation [3.65] which yields the integral equation

$$1 = e^{-ix} \int_{\infty}^{x} e^{i\lambda} k(\lambda) d\lambda \qquad x < 0 \qquad [3.68]$$

At this stage we do not seek a solution of Equation [3.68] by general methods, but adopt an approximate simple expression for $k(z)$ which satisfies only approximately [3.68].

The simplest form of $k(z)$ imaginary along $x > 0$ and having the proper singularity at $z = 0$ is

$$k(z) = \frac{ia}{z^{3/2}} \qquad [3.69]$$

with a an arbitrary constant.

From Equation [3.67] we find

$$f_1(z) = i\ e^{-iz}\ a \int_{\infty}^{z} \frac{e^{i\lambda}}{\lambda^{3/2}}\ d\lambda \qquad [3.70]$$

The integral in [3.70] may be expressed by the aid of the Gamma Incomplete Function (Gradshtein and Rhyzik, 1965) and $f_1(z)$ becomes

$$f_1(z) = ia\ e^{-i(z+\pi/4)}\ \Gamma(-\tfrac{1}{2}, -iz) \qquad [3.71]$$

The function $\Gamma(-\frac{1}{2}, -iz)$ is analytical in the whole z plane cut by $x = 0$, $y > 0$. It has the following asymptotic series (Gradshtein et al, 1965):

(i) For small z:

$$\Gamma(-\tfrac{1}{2}, -iz) = \Gamma(-\tfrac{1}{2}) + e^{i\pi/4} z^{-\frac{1}{2}} \left[2 - \sum_{n=1}^{\infty} \frac{(-1)^n(-iz)^n}{n!\ (n - \frac{1}{2})} \right] \tag{3.72}$$

Hence with $\Gamma(-\frac{1}{2}) = -2\pi^{\frac{1}{2}}$, f_1 has the expansion

$$f_1(z) = -i2\pi^{\frac{1}{2}}\, a\, e^{-i\pi/4} + 2ia\, z^{-\frac{1}{2}} + O(z^{\frac{1}{2}}) \tag{3.73}$$

and

$$\Psi_1(x,0) = -(2\pi)^{\frac{1}{2}}\, a + 2ax^{-\frac{1}{2}} + O(x^{\frac{1}{2}}) \qquad (x > 0)$$

$$\Psi_1(x,0) = -(2\pi)^{\frac{1}{2}}\, a + O(\, x^{\frac{1}{2}}) \qquad (x < 0) \tag{3.74}$$

(ii) For large z:

$$\Gamma(-\tfrac{1}{2}, -iz) = |z|^{-3/2}\, e^{-i3(\arg z - \pi/2)/2}\, e^{iz}\left[1 + O\left(\frac{1}{|z|}\right)\right] \tag{3.75}$$

and

$$f_1 = -a\, e^{-i3 \arg z/2} \left[1 + O\left(\frac{1}{|z|}\right)\right] \tag{3.76}$$

Again, $\Psi_1 = \mathrm{Im} f_1$ follows the expressions

$$\Psi_1 = O\left(\frac{1}{|z|}\right) \qquad (x > 0)$$
$$\Psi_1 = -a + O\left(\frac{1}{|z|}\right) \qquad (x < 0) \qquad [3.77]$$

Unfortunately Ψ_1 is not constant along $x < 0$, as required by Equation [3.65]. But the approximate solution has the proper behavior near the bow, where $w_1 \sim z^{-3/2}$ and $\eta_1 = \Psi_1$ is like $x^{-\frac{1}{2}}$ for $x > 0$, and also at $x \to -\infty$ where $\eta_1 \to -a$ with no waves left behind the body.

Now, it is a matter of convention how to pick the value of a in order to satisfy approximately Equation [3.65]. If we try to satisfy [3.56] near the bow a may be obtained from the condition

$$\Psi_1\ (0,-0) = 1 \qquad [3.78]$$

which gives

$$a = -1/(2\pi)^{\frac{1}{2}} \qquad [3.79]$$

Although we have no exact solution for the outer problem, the approximate expression [3.71] reflects the main features of the solution.

(f) <u>The Matching of the Inner and the Outer Solutions</u>

The matching is generally achieved by an intermediate expansion (Cole, 1968). In the present case it can be done by the simple principle (Van Dyke, 1964): The outer limit of the inner solution equals the inner limit of the outer solution.

Substituting $z = \epsilon^* Z$ in the outer solution [3.71] and seeking the limit $W_1(Z)$ for $\epsilon^* \rightarrow 0$ and $Z = O(1)$ we obtain from Equations [2.16] and [3.73] for the inner limit of the outer solution

$$W = - \frac{ia}{\epsilon^{*1/2} Z^{3/2}} - 1 + O(\epsilon^{*1/2}) \qquad [3.80]$$

The outer solution matches with the outer limit of the inner solution [3.62] only if

$$\Delta \sim \epsilon^{*-1/3} \qquad [3.81]$$

The estimate of [3.81] is the main result of our analysis. In particular, for the value of a of [3.79],

$$\Delta = \frac{\pi^{2/3}}{2} \epsilon^{*-1/3} \qquad [3.82]$$

The matching of η and N_o is also ensured at order $\epsilon^{*-1/2}$ with Δ given by [3.82]: From Equations [2.16] and [3.74] we find for the inner limit of the outer solution

$$N = - \frac{2^{\frac{1}{2}}}{\epsilon^{*\frac{1}{2}} \pi^{\frac{1}{3}} X^{\frac{1}{2}}} + O(1) \qquad [3.83]$$

while the outer limit of the inner solution [3.83] has the form

$$N = - \frac{2^{\frac{1}{2}}}{\epsilon^{*\frac{1}{2}} \pi^{\frac{1}{3}} X^{\frac{1}{2}}} + O(\epsilon^{*-\frac{1}{3}}) \qquad [3.84]$$

(g) Bow Drag

The bow drag is evaluated from the momentum loss in the jet

$$D' = \rho U'^2 \Delta'$$

or, in a dimensionless form

$$D = D'/\rho U'^2 T' = \Delta$$

From Equation [3.81] we have $D \sim Fr_T^{\frac{1}{3}}$ or

$$D' \sim \rho U'^2 T' (U'^2/gT')^{\frac{1}{3}} \qquad [3.85]$$

If we assume that the bow drag for a body of finite length L' has the same expression we have for the bow drag in its conventional form

$$D'/\rho U'^2 L' = (T'/L')(U'^2/gT')^{\frac{1}{3}} = (T'/L')^{\frac{2}{3}}(U'^2/gL')^{\frac{1}{3}} \qquad [3.86]$$

(h) Discussion of Results

In the present section the free-surface flow past a blunt body with high Fr_T number (but low Fr_L) has been studied. The problem is different from that considered in planing studies, since the position of the body is fixed and its bottom is horizontal.

The main results of the analysis are the following:

(i) The proper type of pressure singularity at the bow in the outer solution is of the order $|x|^{-3/2}$. This pressure is not integrable so that lift may be evaluated only via the inner expansion. Obviously, the inner solution shows that the dynamic pressure is a maximum $\rho U'^2/2$ at the stagnation point.

(ii) A jet is assumed to appear at high Fr_T numbers. The energy of the jet is probably entirely dissipated. This is causing a drag additional to the wave resistance.

(iii) The jet thickness and the bow drag grow slowly with Fr_T, like $Fr_T^{1/3}$.

The present analysis may be refined in different directions: By improving the outer solution, by considering bodies of finite length, by studying different bow shapes and by extending the results to real flat ships.

IV. CONCLUSIONS

Special approximations are needed in order to analyze the free surface flow in the vicinity of the bow of blunt ships. In the case of thin ships (T'/B' sufficiently small), the inner bow flow reduces to a two-dimensional flow in a vertical plane normal to the bow. Furthermore, it is appropriate to consider a blunt two-dimensional body of semi-infinite length and this is done herein.

The situation at the confluence of a blunt bow and the free surface is clarified first. It is shown that there are two possible angles between a free surface and a rigid wall at the stagnation point: (i) if the wall is inclined with respect to the horizontal at an angle larger than 120°, the free surface intersects the wall at 120°; (ii) if the wall is inclined at less than 120°, the free surface is horizontal. The latter case is usual for ships.

For small Froude number based on draft, Fr_T, the flow can be analyzed by means of an expansion in Fr_T, according to which the first approximation corresponds to replacing the free surface by a rigid wall. The flow past a rectangular body is analyzed to the second order. The solution to this second approximation shows that the free surface becomes steeper in front of the bow. Application of Taylors instability criterion leads to the conclusion that the stability of the free surface decreases with increasing Fr_T, presumably, culminating in breaking.

In the second approximation, the free surface becomes unstable at Fr_T of about 1.5 and at a position 30 percent of the draft ahead of the bow. The small Fr_T theory does not allow the calculation of bow drag, which only ensues after breaking, but it does permit the estimation of sinkage and trim.

For large Fr_T (but small Fr_L) the flow past a rectangular bow has been analyzed. The problem is different from that considered in planing studies, since the bow is vertical, while the bottom is horizontal. The problem is solved by matching appropriate inner and outer solutions. The inner solution corresponds to a free surface without gravity while the outer flow corresponds to the usual linearized free surface flow with gravity. The main results of the analysis are: (i) the proper type of pressure singularity at the bow in the outer solution if of order $|x|^{3/2}$; (ii) a spray jet appears at the bow, whose energy is probably entirely dissipated. This jet causes a bow drag additional to the usual wave resistance; (iii) the jet thickness and the bow drag grows slowly with Fr_T, like $Fr_T^{1/3}$.

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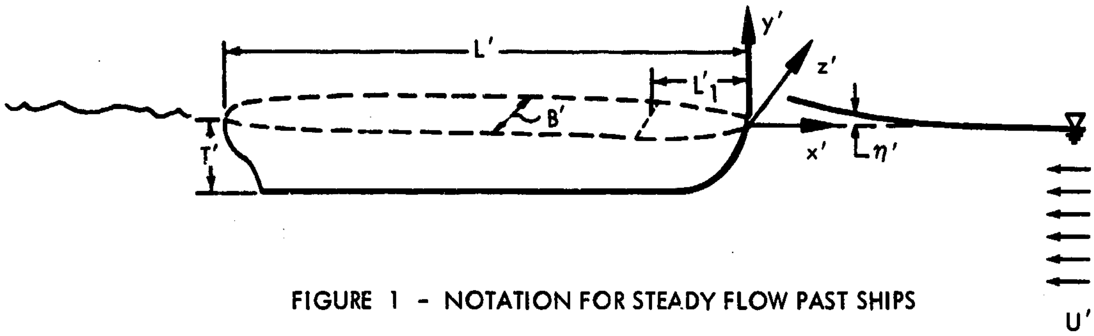


FIGURE 1 - NOTATION FOR STEADY FLOW PAST SHIPS

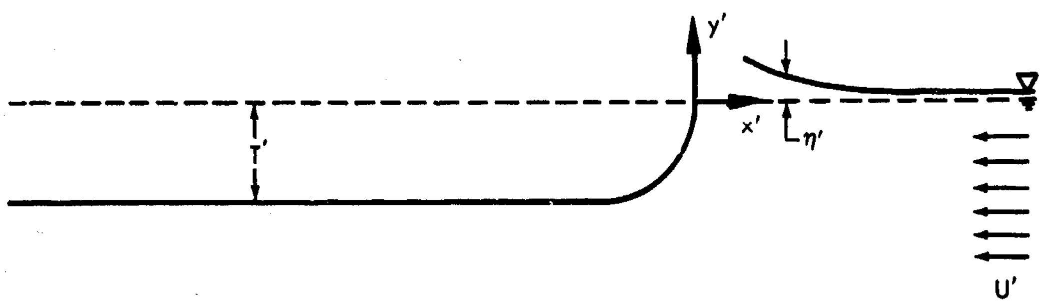


FIGURE 3 - TWO-DIMENSIONAL FLOW PAST A BODY OF SEMI-INFINITE LENGTH

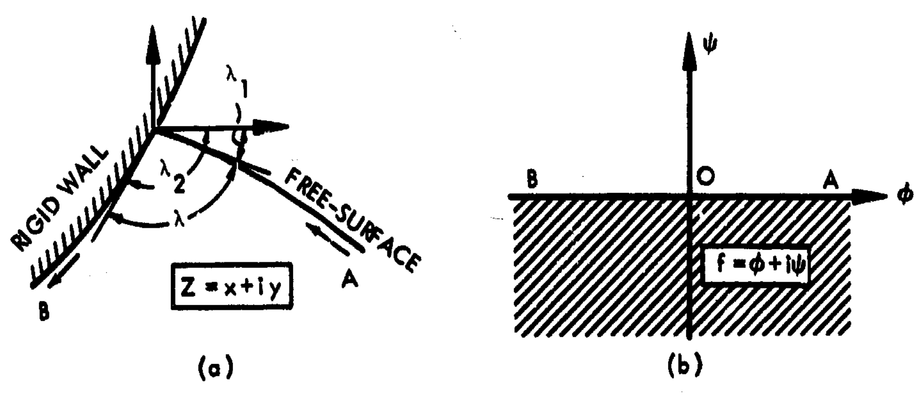
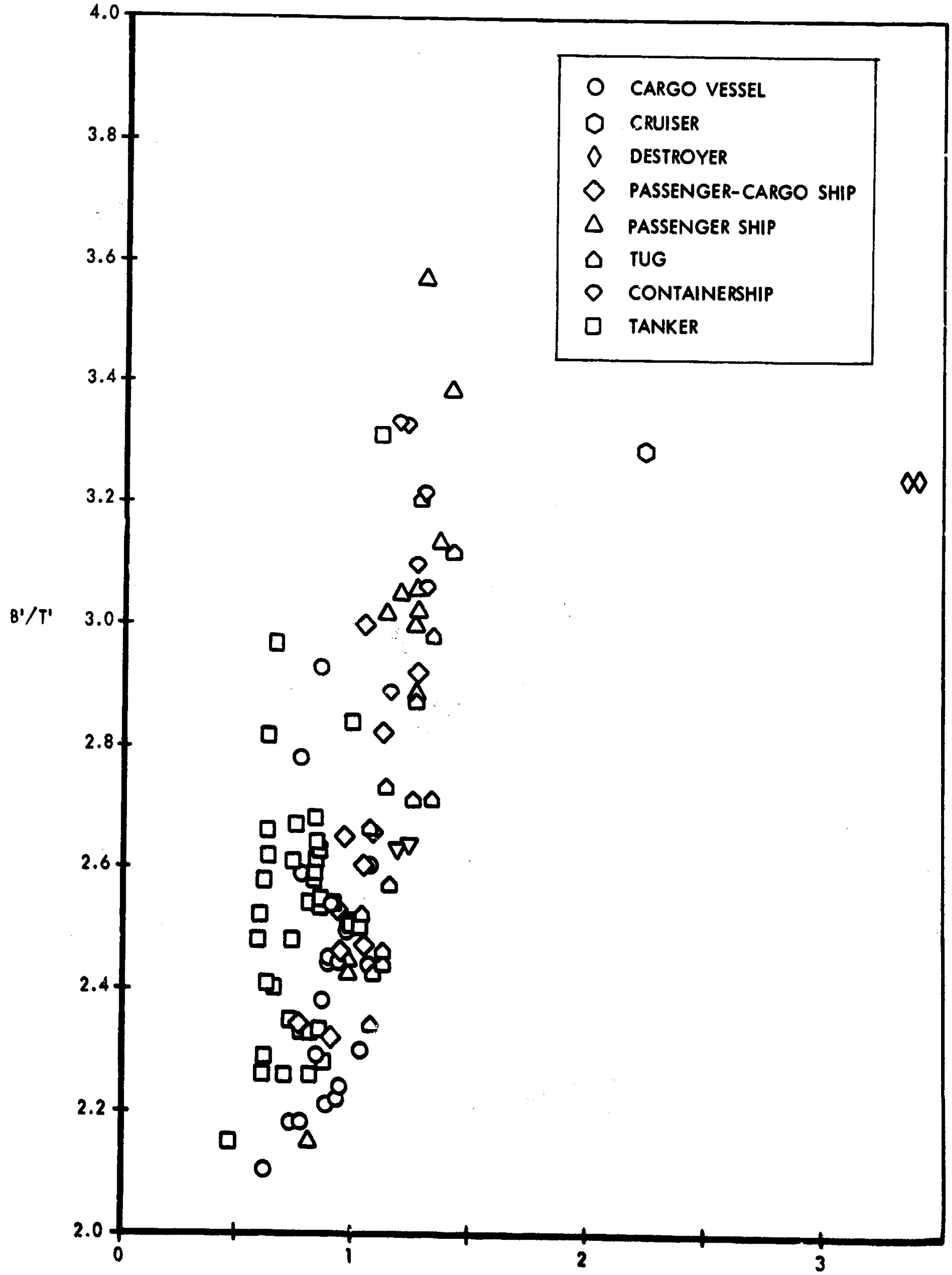


FIGURE 4 - FREE SURFACE FLOW IN THE VICINITY OF A STAGNATION POINT



$Fr_T = U'/(gT')^{\frac{1}{2}}$

FIGURE 2 - BEAM DRAFT VERSUS (FROUDE NUMBER)$_{DRAFT}$

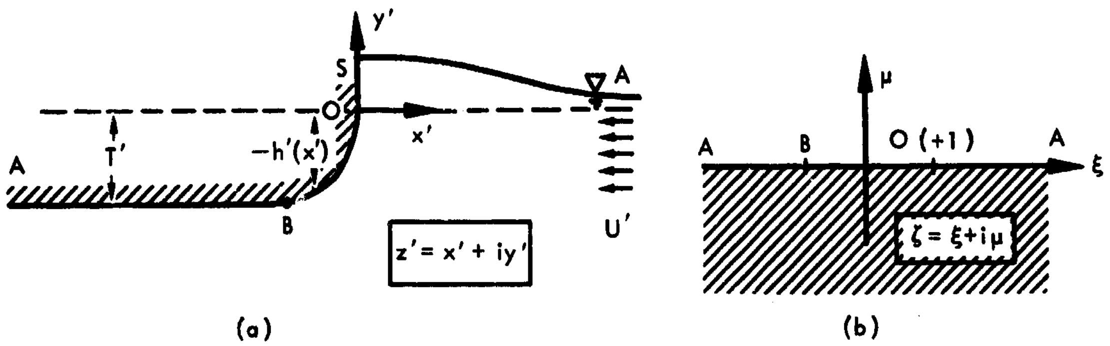


FIGURE 5 - FLOW PAST A BLUNT BODY AT SMALL FROUDE NUMBER

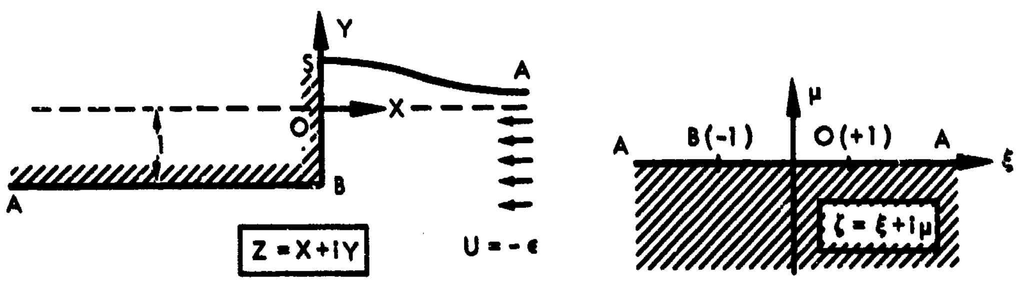


FIGURE 6 - FLOW PAST A RECTANGULAR BODY AT SMALL FROUDE NUMBER

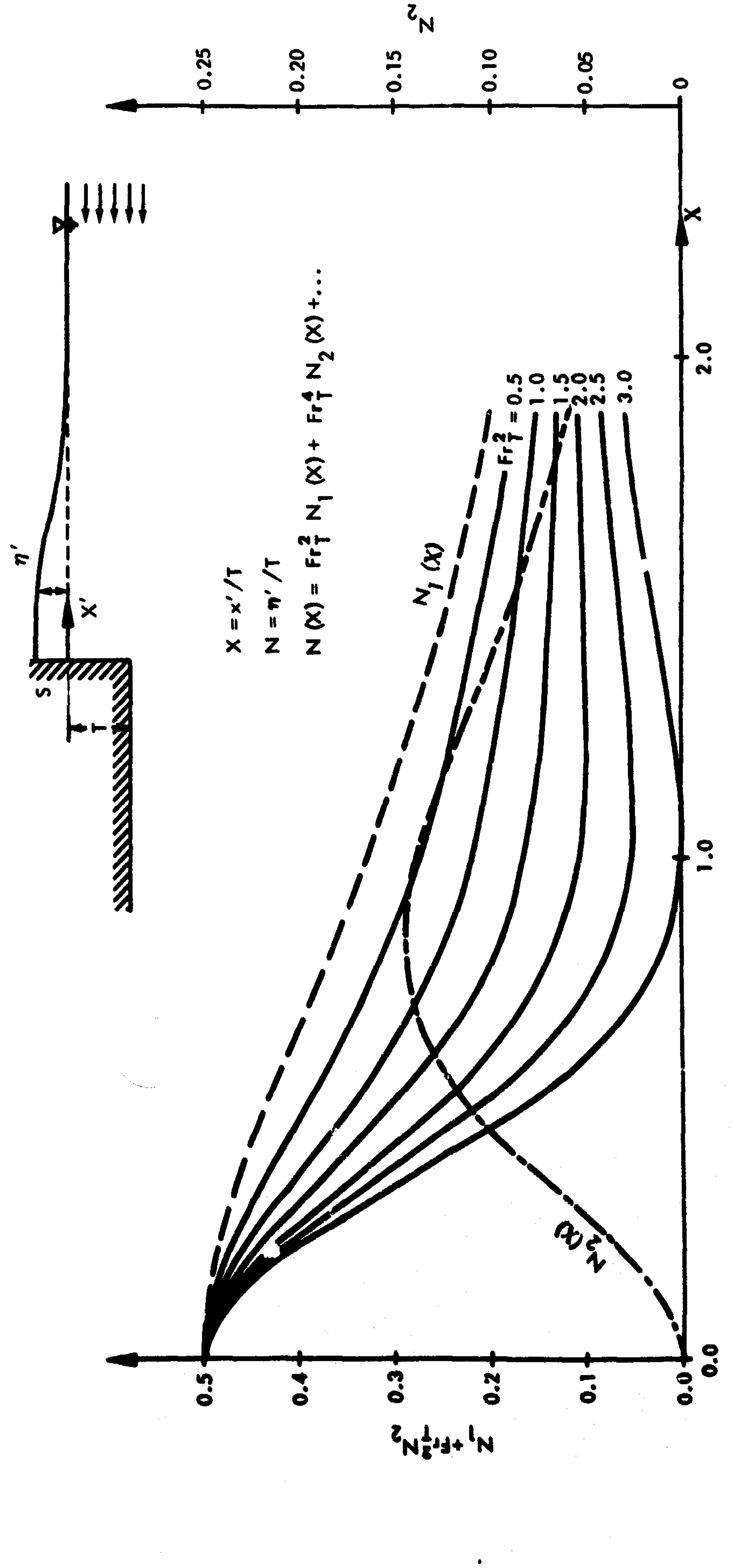


FIGURE 7 - THE FREE-SURFACE SHAPE IN FRONT OF A RECTANGULAR BODY

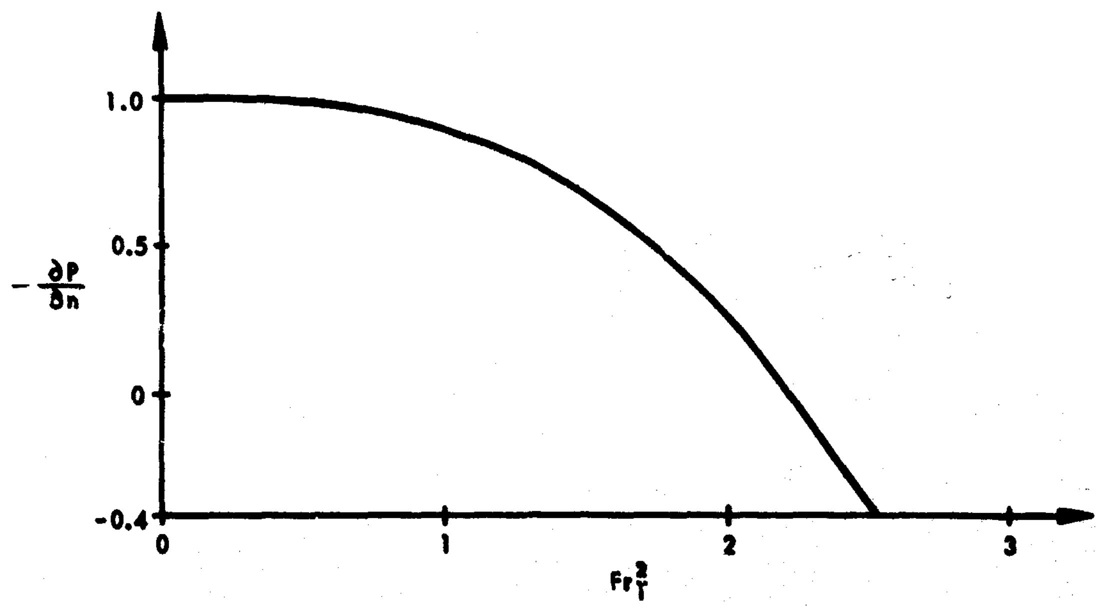


FIGURE 8 - THE RELATIONSHIP BETWEEN THE MINIMUM PRESSURE GRADIENT NORMAL TO THE FREE-SURFACE AND THE FROUDE NUMBER

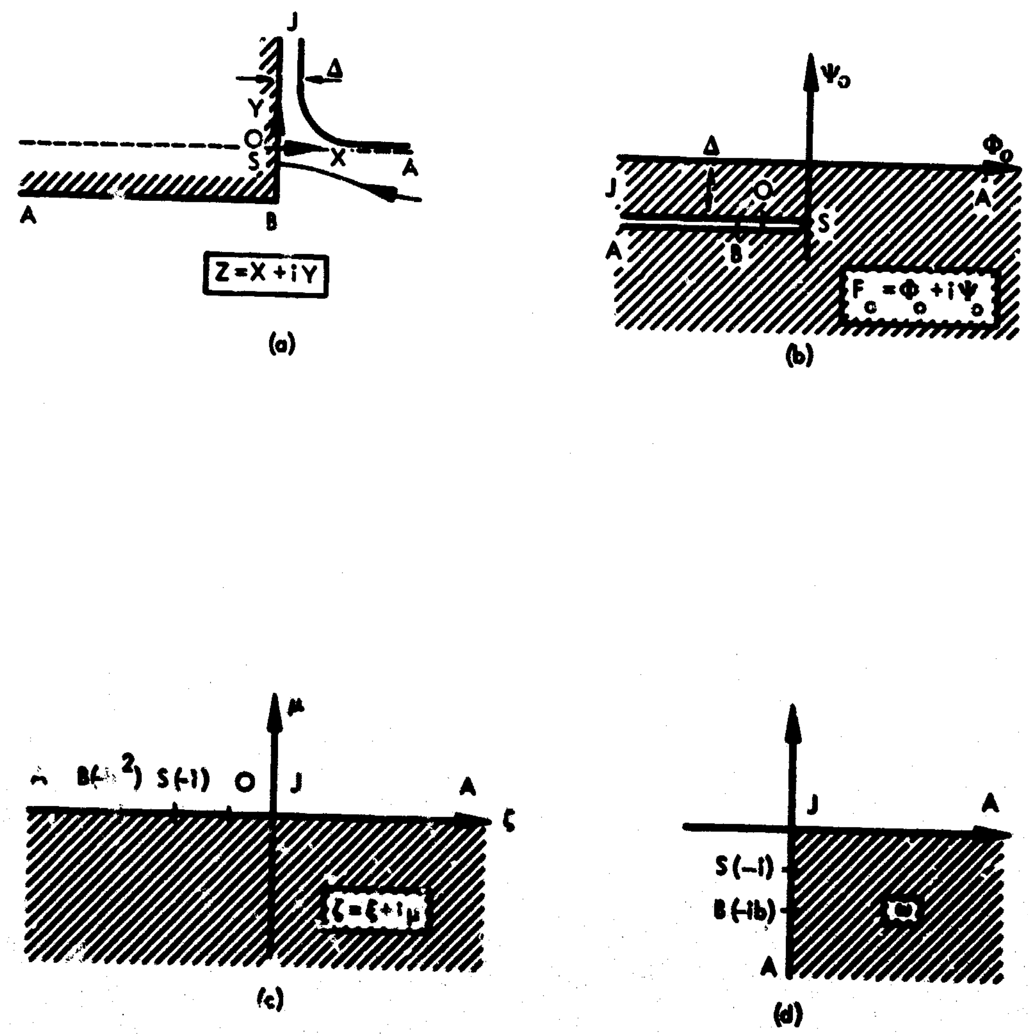


FIGURE 9 - FREE-GRAVITY JET FLOW PAST A RECTANGULAR BODY

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13. ABSTRACT

Special approximations are needed in order to analyze the free surface flow in the vicinity of the bow of blunt ships. In the case of thin ships the inner bow flow reduces to a two-dimensional flow in a vertical plane normal to the bow. The situation at the confluence of a blunt bow and the free surface is clarified first. It is shown that there are two possible angles between a free surface and a rigid wall at the stagnation point: (i) if the wall is inclined with respect to the horizontal at an angle larger than 120°, the free surface intersects the wall at 120°; (ii) if the wall is inclined at less than 120°, the free surface is horizontal. The latter case is usual for ships.

For small Froude number based on draft, Fr_T, the flow can be analyzed by means of an expansion in Fr_T, according to which the first approximation corresponds to replacing the free surface by a rigid wall. The flow past a rectangular body is analyzed to the second order. Application of Taylors instability criterion leads to the conclusion that the stability of the free surface decreases with increasing Fr_T, presumably, culminating in breaking. The small Fr_T theory does not allow the calculation of bow drag, which only ensues after breaking, but it does permit the estimation of sinkage and trim.

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13. ABSTRACT (continuation of Abstract)

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Wave breaking						
Wave theory						

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Page	Line	Actual	Must Be
iii	7	$Fr_T = U'^2/gT'$	$Fr_T = U'/(gT')^{\frac{1}{2}}$
iii	8	$Fr_L = U'^2/gL'$	$Fr_L = U'/(gL')^{\frac{1}{2}}$
iv	8	$\epsilon^* = T'/gU'^2$	$\epsilon^* = T'g/U'^2$
4	5	$\overline{V}$	$\overline{V}'$
5	Eq. (2.15)	$h(x,z) = \epsilon(x,z)$	$h(x,z) = \epsilon H(x,z)$
5	Eq. (2.16)	$\varphi = -x + \epsilon\varphi_1 + O(\epsilon)$	$\varphi = -x + \epsilon\varphi_1 + o(\epsilon)$
		$\eta = \epsilon\eta_1 + O(\epsilon)$	$\eta = \epsilon\eta_1 + o(\epsilon)$
6	9	$\epsilon_B = O(1)$	$\epsilon_B = o(1)$
6	11	$\epsilon = O(1)$	$\epsilon = o(1)$
6	Eq. (2.19)	$v_1 - \eta_{1,x} = 0$	$v_1 + \eta_{1,x} = 0$
6	2 lines from bottom	Equations [2.10]-[2.11]	Equations [2.11]-[2.12]
6	3 lines from bottom	$Fr_L^2 = O(1)$	$Fr_L^2 = o(1)$
12	first line of the table, left	$O(1)$	$o(1)$
16	Eq. (3.4)	(at the end) $\varphi^{(\lambda/\pi-\gamma-2)}$	$\varphi^{(\lambda/\pi+\gamma-2)}$
20	7	Since	since
25	7	In Figure 7a we give ... ... as a function of Fr_T	Delete this sentence
27	7	$T'g/U'^2 \rightarrow (\qquad)$	$T'g/U'^2 \rightarrow o(\qquad)$
32	Eq. (3.55)	$\int_{-ib}^{\omega} \frac{\omega^2 H}{\omega}$	$\int_{-ib}^{\omega} \frac{\omega^2 + 1}{\omega}$
34	Eq. (3.63)	$\frac{1}{x^{-1/2}}$	$-\frac{1}{x^{1/2}}$
38	line 1	Again, $\Psi_1 = Imf_1$ ...	Again, from $\Psi_1 = Imf_1$
Figure 9(b)		(Coordinate)